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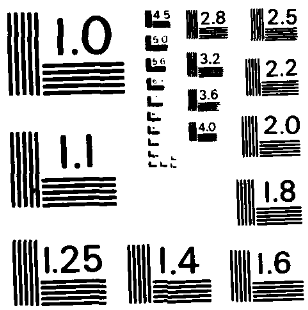

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DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

A RAY THEORY FOR NONLINEAR SHIP WAVES WITH AMPLITUDE INTERACTION

by

Bohyun Yim

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SHIP PERFORMANCE DEPARTMENT
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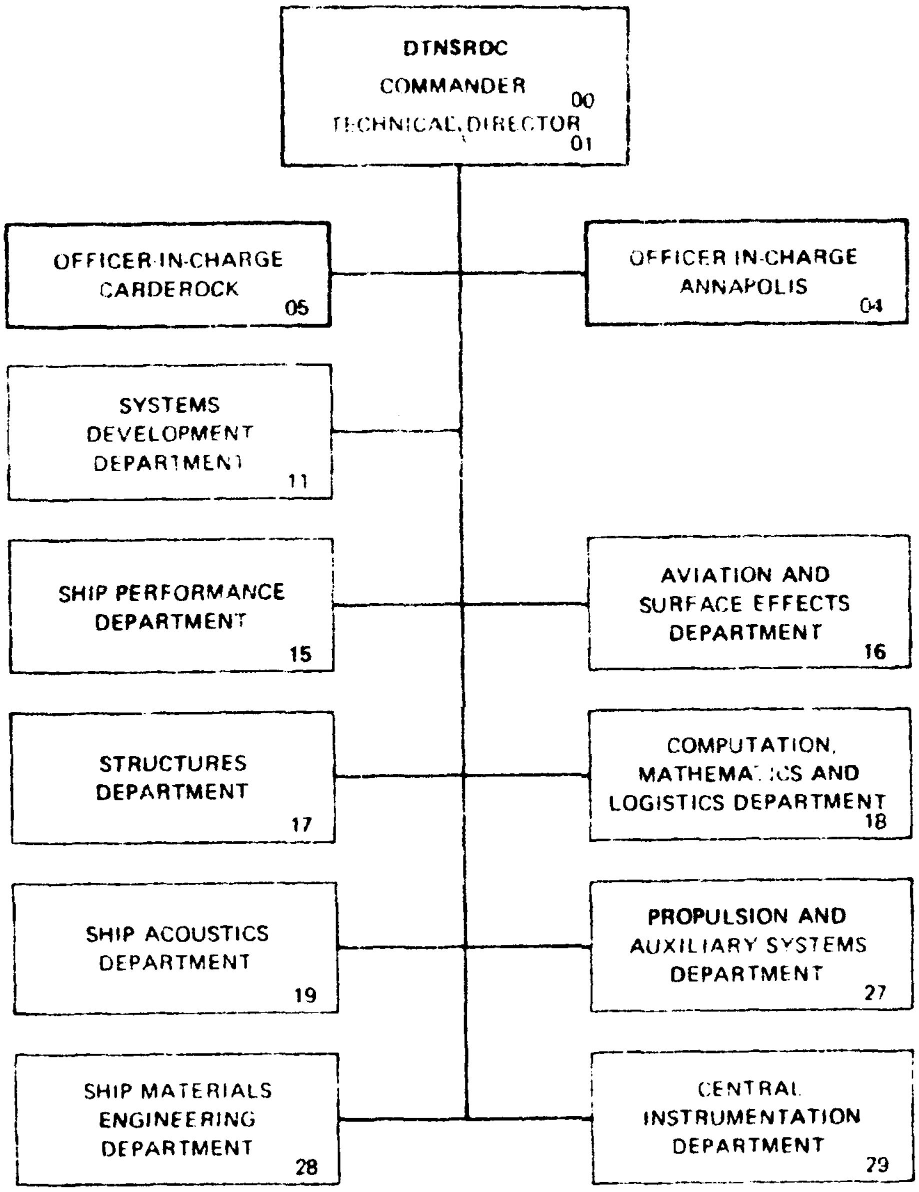

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The effect on ship waves of the nonlinear free surface boundary condition and nonuniform flow are considered. The wave number is represented by a function of space and the wave slope, ak. The value of ak turned out to be larger than the critical value (0.447) in the wide area near the bow, except when the wave crest is nearly perpendicular to the ship hull; fortunately, the latter is the most significant case.

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NOTATION

a	Wave amplitude
b	$= \pi\, b_1$
b_1	Constant defined in Equation (26)
c	Wave speed
F	Function for the free surface given as $z = F(x,y)$
F_n	Froude number
f	Ship shape function given as $y = f(x,z)$
g	Acceleration of gravity
h	Constant defined in Equation (26)
k	*Wave number* $= g/(u \cos \theta + v \sin \theta)^2$
k_o	$= gL/U^2$
L	Ship length
m	Source distribution for a ship
$\bar{n}$	Unit normal vector to wave crest
q	Water particle speed on the free surface relative to a ship
r	Distance between a field point and the origin
s	Wave phase
t	Time
U	Uniform ship speed
u	The x component of flow relative to a ship
u_1	The x component of flow caused by a double model ship relative to the space fixed coordinates
v	The y component of flow relative to a ship

v_1	The y components of flow caused by a double model ship relative to the space fixed coordinates
x,y,z	The rectangular Cartesian coordinates
α	Value defined in Equation (11)
ζ,ζ_r	Function, related to wave height defined in Equation (7)
θ	Angle between $\bar{n}$ and the x axis
θ_∞	The value of θ at $x \to \infty$
ϕ	Potential
ϕ',ϕ_r	Functions related to potential defined in Equation (6)
ω	Wave frequency

ABSTRACT

The effect on ship waves of the nonlinear free surface boundary condition and nonuniform flow are considered. The wave number is represented by a function of space and the wave slope, ak. The value of ak turned out to be larger than the critical value (0.447) in the wide area near the bow, except when the wave crest is nearly perpendicular to the ship hull; fortunately, the latter is the most significant case. Yim's previous work on ray theory is used to include the full nonlinear effect; the ray and the wave phase of a Wigley hull are computed by the present method and are compared with previous results. When compared to the nonuniform flow effect, the nonlinear free surface effect increases further both the Kelvin wedge angle near the bow and the difference of wave phase from the predictions of linear theory.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

Researchers have advanced two-dimensional nonlinear wave theory considerably since Stokes[1*] represented the wave height by a series of products of wave-amplitudes and wave numbers. Lonquet-Higgins,[2] Schwatz,[3] Cokelet,[4] and many others have contributed to developing accurate numerical predictions of two-dimensional wave forms up to the breaking stage. Having assumed that the wave number was slowly varying in time and space compared with the wavelength, Whitham[5] analyzed three-dimensional waves using the two-dimensional theory of nonlinear waves. Using Whitham's idea, Lighthill[6] also contributed significantly to an understanding of three-dimensional water waves.

In a discussion of a paper by Gadd[7] on ship waves, Lighthill suggested that the Whitham concept could be used to find a nonlinear correction to ship waves. When Hogben[8] used nonlinear theory to determine phase relations for ship waves, which

*A complete listing of references is given on page 29.

were quite different from the results of linear theory, he encountered difficulties in following Lighthill's advice. Following a different approach, Hogben applied two-dimensional Fourier expansions with a large number of Fourier coefficients to represent water waves caused by a simple submerged source. The Fourier coefficients were determined so as to satisfy the exact free surface boundary conditions. Hogben then obtained wave phases for various submergences, source strengths, and Froude numbers.

Yim[9] recently applied a ray theory of ship waves and found the predicted wave phase to differ from the linear theory. The difference was caused by the nonuniform flow created by the ship itself. In ray theory a ray is the path of a wave energy packet from the wave source such as the ship bow. In the conventional linear ship-wave theory a ray is a straight line so that, in relation to wave propagation, linear theory neglects the flow perturbation created by an advancing ship. However, if the flow perturbation is considered using even a double model hull representation, the ray is curved, and some rays that pass near the ship have to reflect from the ship boundary. The Yim analysis considered the linear free surface condition as modified by the nonuniform flow. The dispersion relation was linear but modified to account for the nonuniform flow, as done by Ursell[10] in 1960. A fully nonlinear free-surface condition includes not only the nonuniform flow effect but also the interaction between wave numbers and wave amplitudes.[1]

The present paper considers both the effect of the nonlinear free surface condition on ship waves and the effect of the nonuniform flow. The method is similar to that suggested by Lighthill.[6] The wave number is represented by a function of space and the wave slope, ak. The value of ak turns out to be larger than the critical value (0.447) in a wide region near the bow, except when the wave crest is nearly perpendicular to the ship hull; fortunately, the latter is the most significant case. Because of Yim's previous work on ray theory, it was not difficult to include the full nonlinear effect. The ray and the wave phase of the Wigley ship hull[9] are computed by the present method and are compared with the previous results. For the first time, the Whitham[5] nonlinear wave theory is applied to the actual ship model.

BOUNDARY CONDITIONS FOR SHIP WAVE PROBLEMS

A right-handed rectangular coordinate system is used, with z upward, x and y on the mean free surface with the origin fixed in space. At any time the fluid is considered to be homogeneous, inviscid, irrotational, and infinitely deep. When a ship advances with a uniform speed -U in the negative x direction, there exists a velocity potential ϕ that satisfies

$$\nabla^2\phi = 0 \tag{1}$$

with velocity $-\nabla\phi$;

$$-\nabla\phi = u\bar{i} + v\bar{j} + w\bar{k} \text{ and } -\nabla\phi = 0 \text{ at infinity}$$

The boundary conditions for ϕ on the free surface, $z=F(x,y,t)$, are:

$$F_t - \phi_x F_x - \phi_y F_y + \phi_z = 0 \tag{2}$$

$$F(x,y) + \frac{1}{2g} (\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2 - 2\phi_t) = 0 \tag{3}$$

w' .e g is the acceleration of gravity.

As a simple approximation, all the quantities perturbed by the presence of a ship in otherwise uniform flow are considered sufficiently small so that the second-order terms of Equations (2) and (3) are negligible. Then Michell's linear boundary conditions are obtained. When F is eliminated between Equations (2) and (3), the well-known linear free-surface condition is obtained,

$$\phi_{tt} + g\phi_z = 0 \tag{4-1}$$

If the coordinate is fixed at the ship bow, Equation (4-1) becomes

$$\phi_{xx} + \frac{g}{U^2}\phi_z = 0 \tag{4-2}$$

The boundary condition on the ship surface $y = f(x,z)$ is:

$$(U-\phi_x)\, f_x + \phi_y - \phi_z\, f_z = 0 \tag{5}$$

The Michell thin ship approximation has been studied extensively for ship waves and wave resistance and has helped promote an understanding of the related physics. However, the Michell theory can predict neither wave heights nor the wave resistance accurately, except when the ship is thin. The Michell solution indicates that the effects of beam-length ratio B/L and Froude number $F_n = U/\sqrt{gL}$ are important and complex, such that problems involving high-speed ships should be treated separately from those for slow-speed ships. Also, the shape of the ship near the free surface affects the solution more than it does near the bottom of the ship, especially for slow-speed ships.

For a steady slow-ship theory, the perturbation potential ϕ and the free surface elevation F are written as

$$\phi = \phi_r + \phi' \tag{6}$$

$$F = \zeta_r + \zeta \tag{7}$$

where ϕ_r and $\zeta_r = (U^2-\phi_{rx}^2-\phi_{ry}^2)/(2g)$ are caused by a double model representation of the hull.

Using the order of magnitude assumptions:

$$\phi' = O(U^5),\ \zeta = O(U^4)$$

and for ϕ' and ζ,

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = O(U^{-2}) \tag{8}$$

and for ϕ_r and ζ_r $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = O(1)$; $\phi_r = O(U)$, $\zeta_r = O(U^2)$.
The lowest-order free-surface condition for slow ships to $O(U^3)$ has been derived by Ogilvie[11] as

$$\frac{1}{g}\left\{\phi_{rx}(x,y,o)\frac{\partial}{\partial x} + \phi_{ry}(x,y,o)\frac{\partial}{\partial y}\right\}^2 \phi'(x,y,z) + \phi'_z(x,y,z) = D(x,y)$$

$$\text{on } z = \zeta_r \tag{9}$$

where

$$D(x,y) = \frac{\partial}{\partial x}\{\phi_{rx}(x,y,o)\zeta_r(x,y)\} + \frac{\partial}{\partial y}\{\phi_{ry}(x,y,o)\zeta_r(x,y)\} \tag{10}$$

DISPERSION RELATION

The Laplace equation is satisfied by the potential for an elementary wave whose normal to the crest makes an angle θ with the x axis,

$$e^{kz+i\alpha} \equiv e^{kz+ik(x\cos\theta+y\sin\theta)-i\omega t} \tag{11}$$

where ω is the frequency and k is the wave number.

Substituting Equation (11) in the free surface boundary condition, Equation (4-1), the well-known dispersion relation,

$$\omega^2 - gk = 0 \tag{12}$$

can be obtained.

Because the wave form relative to a ship moving at constant velocity -U is stationary, the wave speed c, with respect to the calm water surface, should be $c = U\cos\theta$ so that

$$\omega \equiv ck = Uk\cos\theta \tag{13}$$

This relation is not a result of linearization but an exact relation. From Equations (12) and (13) the wave number is given by, $k = g/(U \cos \theta)^2$.

When the local flow field at a point is considered using a local coordinate system for which the local velocity at the point is zero, the relative frequency ω_r is not necessarily the same as ω. In Equation (11), Whitham[5] assumed that k and ω are slowly varying compared to the local wavelength, and showed that

$$\bar{k} = k_1 \bar{i} + k_2 \bar{j} \tag{14}$$

$$\begin{aligned} \omega_r &= \frac{\partial \alpha}{\partial t} + (u_1 \bar{i} + v_1 \bar{j}) \nabla \alpha \\ &= \omega + u_1 k_1 + v_1 k_2 = k\{(U+u_1)\cos \theta + v_1 \sin \theta\} \end{aligned} \tag{15}$$

$$|\bar{k}| = k$$

where (u_1, v_1) are the local flow field assumed to be caused by a double model with respect to the original coordinate system fixed in space. The values of u_1 and v_1 also vary slowly. Equation (12) is the well-known linear dispersion relation in the absence of a ship. When ship singularities are present the linear dispersion relation should hold[6] for the relative frequency ω_r. That is, from Equations (12) and (15)

$$\omega_r = \sqrt{kg} = k\,\{(U+u_1)\cos \theta + v_1 \sin \theta\} \tag{16}$$

from which the wave number is given by

$$k = g/\{(U+u_1)\cos \theta + v_1 \sin \theta\}^2 \tag{17}$$

This is a slowly varying wave number that satisfies the linear dispersion relation but includes the effect of nonuniform flow. This dispersion relation can also be obtained from the slow-ship free surface condition, Equation (9).

Stokes[1] considered nonlinear two-dimensional water waves and represented the frequency as a series in $(ak)^2$ where a is the wave amplitude and k is the wave number. Lighthill[12] suggested using a closed form

$$\omega_r^2 = gk(1+a^2k^2) \tag{18}$$

as a good approximation for a nonlinear dispersion relation because this is almost exact for both critical wave slope at $(ak)^2 = 0.2$ and a very small wave slope. The wave number may be obtained from Equations (15) and (18) and is given by

$$k = \left[1+a^2g^2/\{(U+u_1)\cos\theta+v_1\sin\theta\}^4\right]g/\{(U+u_1)\cos\theta+v_1\sin\theta\}^2$$

$$= k_\ell(1+a^2k_\ell^2) \tag{19}$$

with $k_\ell \cong g/(u\cos +v\sin)^2$, $u = U + u_1$, $v = v_1$ accurate up to the order of $a^2k_\ell^2$.

In Equation (19), k is the slowly varying wave number that satisfies the nonlinear dispersion relation in nonuniform flow. Although the exact free-surface condition should be satisfied on the free surface and not on the mean free surface, it is noteworthy that the wave number in Equation (19) is represented by the double model velocity field and the amplitude relative to the mean free surface, z=0.

As can be seen by taking the gradient of the potential of an elementary wave, identity Equation (11), derivatives with respect to x, y and z involve a multiplication factor $kL = O(gL/U^2) = O(F_n^{-2})$, which is a large number for slow ships. Therefore, the derivatives of flow quantities with respect to x, y or z change the order of magnitude. This is the main reason for treating slow ships separately from fast ships. Eggers[13] treated the free-surface condition for fast ships and derived a higher-order, free-surface condition on z=0, which is quite different from Equation (9). Then he obtained a different dispersion relation which indicated a region that does not allow waves near the bow.

RAY EQUATION

When the wave number k is known as a function of space, the solenoidal nature of the wave number vector given in Equation (14) leads to the ray equation

$$\nabla \times k = 0 \tag{20}$$

where $\bar{k} = k \cos \theta\ \bar{i} + k \sin \theta\ \bar{j}$.

Rays for the wave number that satisfies the linear dispersion relation, and includes the effect of nonuniform flow, have been investigated in detail by Yim.[9] The nonuniform flow effect curves rays near the ship. This is especially marked when the wave crest is nearly perpendicular to the ship surface. The ray refracts toward the ship and reflects from the ship surface to form the second caustic.

When Adachi[14] analyzed experimental wave height spectra, he found that the measured amplitude spectra were lower in value than predicted by linear theory, over the whole range of θ, and much lower at values of θ less than 20 degrees. The sheltering effect[15] has been shown to reduce wave resistance 15-25% below that predicted by linear theory, which is consistent with reduced wave amplitudes. However, it was not known why the amplitudes were particularly reduced at values of θ less than 20 degrees. If the wave reflection of elementary waves for 20 degrees is considered, the experimental phenomena can be understood. Although the flow field near the ship may be further illuminated experimentally by comparing a long wedge type of ship bow, which does not allow ray reflection, and a rounded bow which induces multiple ray reflections.[9] A bow bulb reduces the nonuniform flow and also prevents reflection if a proper size bulb is used.[16]

When only the conventional linear dispersion relation in regard to wave number is considered without the nonuniform flow effect, all the rays will be straightlines, and the ray theory adds no useful information to that obtained by applying the linear wave theory. To obtain useful information, researchers must consider nonuniform flow and/or a nonlinear dispersion relation. Although ray theory with a nonlinear dispersion relation has never been considered for ray paths, it may not be too difficult to include the effects of both nonlinear dispersion and nonuniform flow.

NONLINEAR RAYS

From Equation (20)

$$\frac{\partial k_1}{\partial y} - \frac{\partial k_2}{\partial x} = 0 \tag{21}$$

where $k_1 = k \cos\theta$, $k_2 = k \sin\theta$

or

$$k_y \cos\theta - k_x \sin\theta - k \sin\theta\, \theta_y - k \cos\theta\, \theta_x = 0 \tag{22}$$

Differentiation of Equation (19) yields

$$\begin{aligned} k_x &= (1+3a^2k^2)\, k_{\ell x} + 2\, a\, k_\ell^3\, a_x \\ k_y &= (1+3a^2k^2)\, k_{\ell y} + 2\, a\, k_\ell^3\, a_y \end{aligned} \tag{23}$$

and substituting $k_{\ell x}$ and $k_{\ell y}$ from Equation (19) into Equation (22), we have

$$\begin{aligned} &\{(1+a^2k_\ell^2)\sin\theta(u\cos\theta+v\sin\theta)+2(1+3a^2k_\ell^2)\cos\theta(-u\sin\theta+v\sin\theta) \\ &-2ak_\ell^2 a_\theta \cos\theta(u\cos\theta+v\sin\theta)\}\frac{\partial\theta}{\partial y} + \{(1+a^2k_\ell^2)\cos\theta(u\cos\theta+v\sin\theta) \\ &-2(1+3a^2k_\ell^2)\sin\theta(-u\sin\theta+v\cos\theta)+2ak_\ell^2 a_\theta \sin\theta(u\cos\theta+v\sin\theta)\}\frac{\partial\theta}{\partial x} \\ &=2(1+3a^2k_\ell^2)\sin\theta\left(\cos\theta\frac{\partial u}{\partial x}+\sin\theta\frac{\partial v}{\partial x}\right) \\ &-2(1+3a^2k_\ell^2)\cos\theta\left(\cos\theta\frac{\partial u}{\partial y}+\sin\theta\frac{\partial v}{\partial y}\right)-2ak_\ell^2\left(\sin\theta\frac{\partial a}{\partial y}-\cos\theta\frac{\partial a}{\partial y}\right) \\ &\cdot(u\cos\theta+v\sin\theta) \end{aligned}$$

This first order partial differential equation for θ is equivalent to the simultaneous ordinary differential equations

$$\frac{dx}{dt} = (1+a^2k_\ell^{\,2}) \cos \theta(u \cos \theta+v \sin \theta) -2(1+3a^2k_\ell^{\,2}) \sin \theta(-u \sin \theta+v \cos \theta)$$

$$+ 2 a k_\ell^{\,2} a_\theta \sin \theta(u \cos \theta+v \sin \theta)$$

$$\frac{dy}{dt} = (1+a^2k_\ell^{\,2}) \sin \theta(u \cos \theta+v \sin \theta)$$

$$+ 2(1+3a^2k_\ell^{\,2}) \cos \theta(-u \sin \theta+v \cos \theta) -2 a k_\ell^{\,2} a_\theta \cos \theta(u \cos \theta+v \sin \theta)$$

$$\frac{d\theta}{dt} = 2(1+3a^2k_\ell^{\,2}) \sin \theta \left(\cos \theta \frac{\partial u}{\partial x} + \sin \theta \frac{\partial v}{\partial x}\right) \tag{24}$$

$$- 2(1+3a^2k_\ell^{\,2}) \cos \theta \left(\cos \theta \frac{\partial u}{\partial y} + \sin \theta \frac{\partial v}{\partial y}\right)$$

$$- 2 a k_\ell^{\,2} \left(\sin \theta \frac{\partial a}{\partial x} - \cos \theta \frac{\partial a}{\partial y}\right) (u \cos \theta+v \sin \theta)$$

with initial conditions placed on x, y, and θ.

At present, there is no other choice of wave amplitude, a, except that of linear theory or that of the linear theory modified by the sheltering effect.[15] Because in the ray equation the amplitude, a, appears only as a^2, the linear amplitude may be acceptable, except near the first caustic at the Kelvin wedge, where a is singular. The linear wave number k_ℓ is very large near the ship bow where the stagnation point is located. Thus, the region where $(ak)^2 \geq 0.2$ will be near the bow and near the first caustic. Because there is no two-dimensional wave with values of $(ak)^2$ larger than the critical value of 0.2 with the present theory, it is impossible to consider nonlinear ship waves with $(ak)^2 \geq 0.2$.

The linear wave amplitude far from the wave source can be obtained by the method of stationary phase,[17] where the amplitude is a function of θ and proportional to

r^{-1}; r represents the distance between the field point and the wave source. Because the wave action contained in a ray tube is unchanged, when the far field wave energy and the ray paths are known the wave energy can be obtained. Wave action is the wave energy divided by the relative frequency, and the wave energy is a function of wave amplitude.

The wave phase s can be obtained in the same way as for only the nonuniform flow effect by

$$ds = \bar{k} \cdot d\bar{r} = k_\ell \ (1+a^2 k_\ell^{\ 2}) \cos\theta \ dx + k_\ell \ (1+a^2 k_\ell^{\ 2}) \sin\theta \ dy \qquad (25)$$

However, when $(ak)^2 \geq 0.2$, the wave breaks and the physics of the wave propagation and the energy relation may change. Nevertheless, the maximum phase change can be obtained by simply using a value of 0.2 for $(ak)^2$ in Equation (25), which yields 1.2 times the value of the phase obtained when only the nonuniform flow is considered.

When the linear wave amplitude is used to solve Equation (24), the choice of initial values of (x,y,θ) poses a serious problem. Previous experience has shown that, given the nonuniform flow effect, many rays are outside of the first caustic predicted by linear theory, where no linear wave is supposed to exist. In addition, the value of linear wave amplitude is infinite at the first caustic. In both linear and nonlinear theories the wave amplitudes at $x \to \infty$ for the same value of θ_∞ may be assumed to be about the same for the present approximation. However, in the present nonlinear theory rays at $x \to \infty$ can be obtained only when the approximate amplitude is known all the way to $x \to \infty$ from the initial point near the origin. Therefore, rays due to nonuniform flow only, without amplitude interaction, were computed first together with the value of θ_∞.

The θ_∞ value was then used to compute linear wave amplitudes by the stationary phase method. Because rays are assumed to start from a point wave source, their amplitudes, as obtained by either the energy method or the stationary phase method, are singular at the initial point or at the wave source. The stationary phase method is used for the asymptotic value of the amplitude at large x values. However, if rays are considered to be straight lines, the same value of amplitude near the wave source results from either the stationary phase method or the energy method for a constant wave number. Therefore, in the present analysis the stationary phase

method was applied in all cases to obtain the amplitudes and their derivatives for use in computing nonlinear ray paths and θ. The final θ value at $x \to \infty$ for the nonlinear ray paths may be different from θ_∞ which is used for the linear wave amplitude. However, the linear amplitude changes slowly at points not too near the wave source for small θ_∞ values where the present calculation of nonlinear ray paths is meaningful. Therefore, errors of higher order may be induced by using the approximate values of wave amplitude. Some numerical results given below for a Wigley hull seem to indicate that this conjecture is valid.

EXAMPLES AND DISCUSSION

Using Equation (24), nonlinear rays are computed for a Wigley hull whose source distribution is represented by

$$m = b_1 \; (1-2x) \left[1- \left(\frac{z}{h}\right)^2\right] \text{on } 0 < x < 1, \; -h < z < h \qquad (26)$$

Appendix A contains equations for the wave amplitudes and their derivatives for the Wigley hull. Equations for the flow field to be used in the ray equation are given in Appendix B. These equations are exactly the same as those in Reference 9 when several misprints in Reference 9 have been corrected.

In Figures 1 through 3, ray paths computed according to Equation (24) are compared with earlier results derived by Yim[9] in which only the nonuniform flow effect was considered without the effect of amplitude interaction. Inclusion of the amplitude interaction seems to push the ray away from the ship when $0 \; |\theta_\infty| < 35$ deg (0.61 radians) just as the nonuniform flow effect did to the conventional linear ray theory. When $|\theta_\infty| > 35$ deg (0.61 radians), the amplitude interaction effect pulls the ray toward the ship.

In Figure 3 the ray paths for $\theta_\infty = -0.307$ and -0.292 radians are almost identical, although the linear amplitudes are used for $\theta_\infty = -0.246$ and -0.0909 radians, respectively. This means that the linear amplitudes for computation of the nonlinear ray theory are reasonably insensitive to θ_∞ for small θ_∞ values.

In Figures 4 through 6, $s_2(\theta)$ denotes the radian phase difference[9] divided by k_o of waves as predicted by the ray theory and linear theory. At far down stream the wave number is constant, and for $\theta \simeq 0$ the wave number equals k. Therefore,

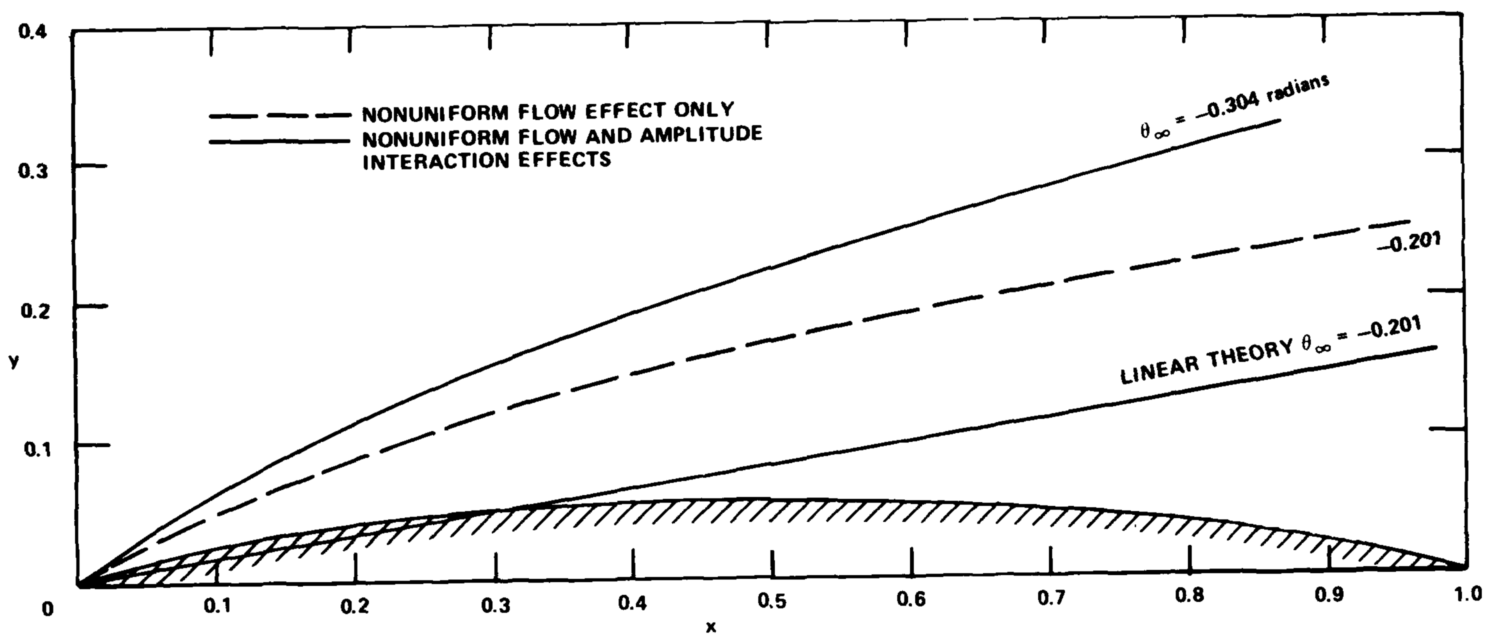

Figure 1 - Effects of Amplitude Interaction on Ray Paths of the Wigley Hull
($b=0.2$, $h=0.0625$, $F_n=0.25$)

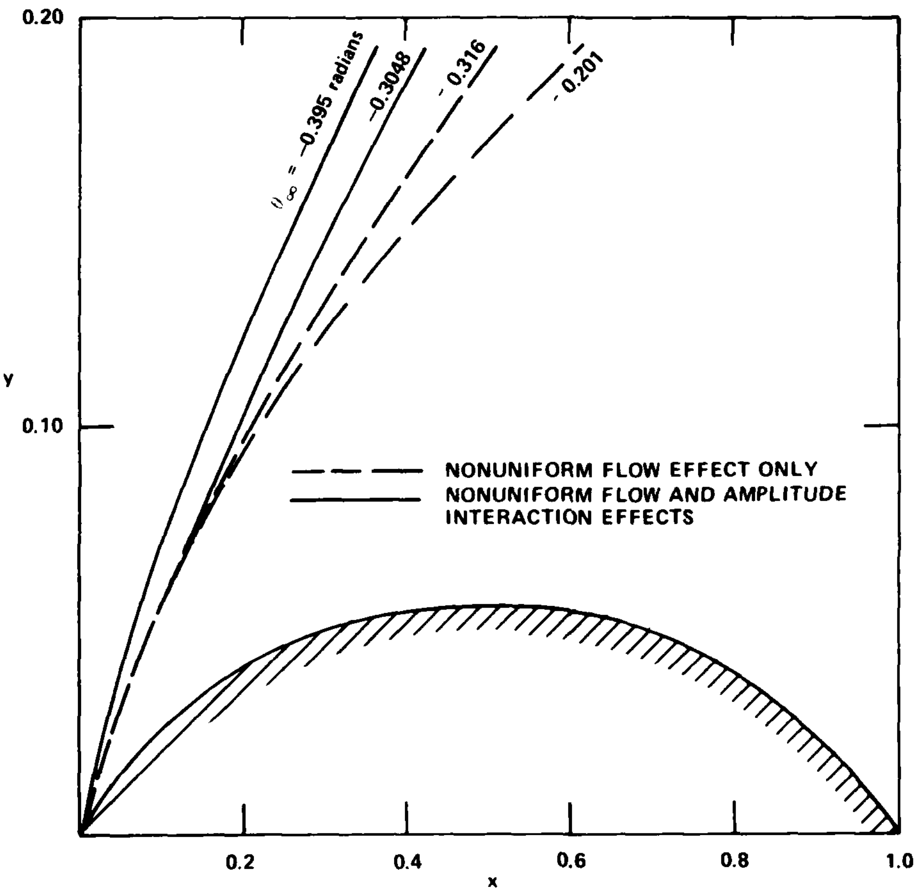

Figure 2 - Effects of Amplitude Interaction on Ray Paths of the Wigley Hull ($b=0.2$, $h=0.0625$, $F_n=0.25$); Figure 1 with an Expanded Vertical Scale

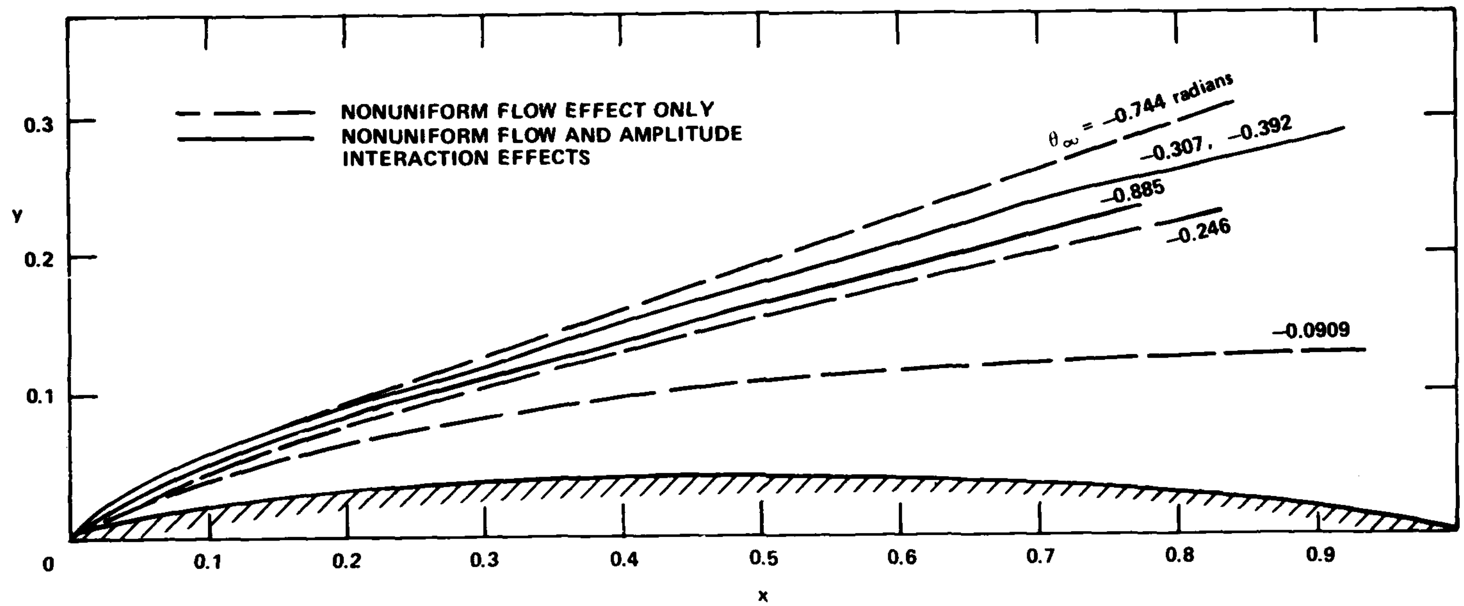

Figure 3 - Effects of Amplitude Interaction on Ray Paths of the Wigley Hull
($b=0.2$, $h=0.03$, $F_n=0.25$)

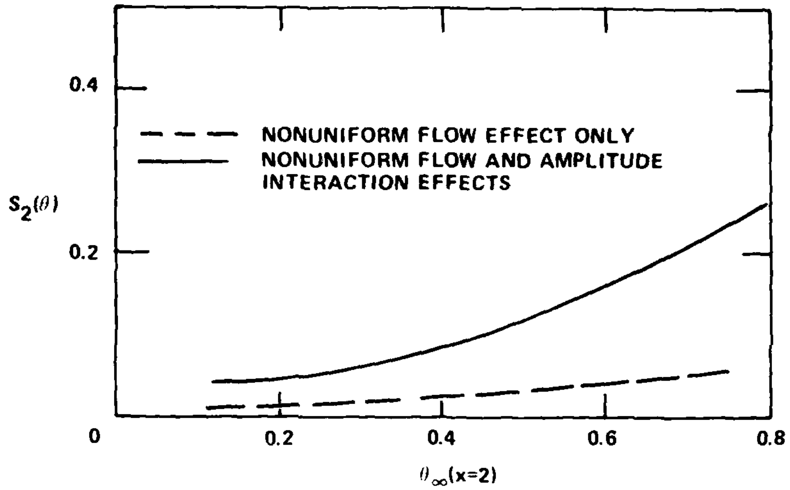

Figure 4 - Phase Difference, $s_2(\theta)$, between Linear and Nonlinear Theory of the Wigley Hull ($b=0.2$, $h=0.0625$, $F_n=0.25$) at $x=2$

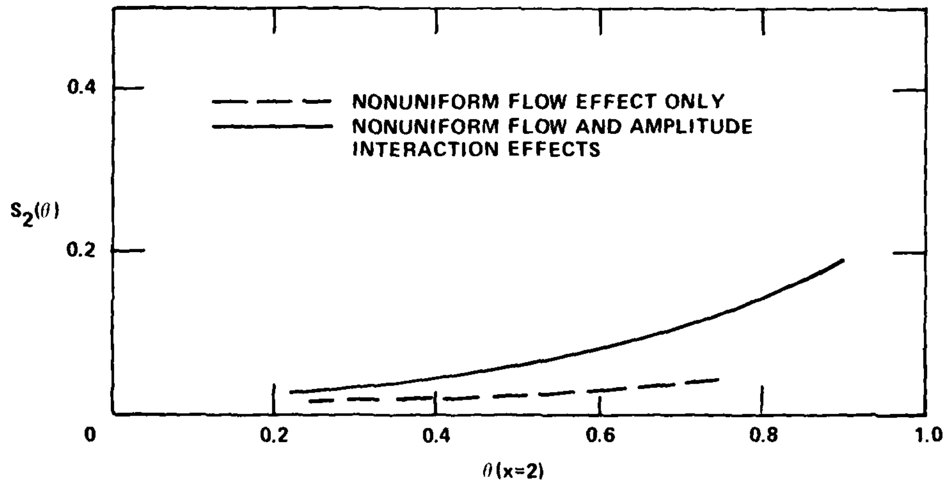

Figure 5 - Phase Difference, $s_2(\theta)$, between Linear and Nonlinear Theory of the Wigley Hull ($b=0.2$, $h=0.03$, $F_n=0.25$) at $x=2$

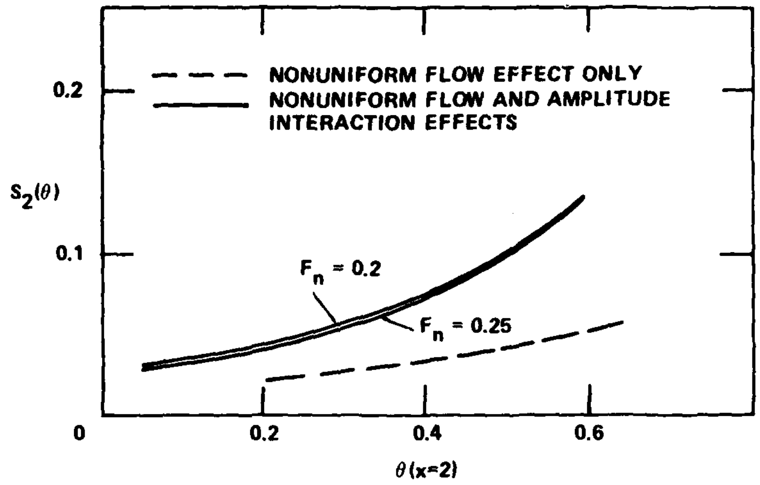

Figure 6 - Phase Difference, $s_2(\theta)$, between Linear and Nonlinear Theory of the Wigley Hull (b=0.2, h=0.0625, F_n=0.28 and 0.25) at x=0.5

$s_2(\theta)$ may be considered to be the fractional wavelength nondimensionalized by the ship length converted from the wave phase difference because the wavelength is $2\pi/k_o$. As Figures 4-6 show, the effect of amplitude interaction on the wave phase is considerably larger than the effect of nonuniform flow. However, the total magnitude of the phase differences from the linear theory seems to be quite close to experimental results[18] showing phase differences of 3-5% of the ship's length (L) along the ship hull.

Hogben[8] calculated the phase difference between the nonlinear and linear waves caused by a submerged point source. Because the case involved a submerged body, he was able to demonstrate the phase advancement in the nonlinear theory without having to consider the sheltering effect. In the present theory the phase advancement was clarified further by separating it into contributions caused by amplitude interaction and by nonuniform flow. In Figure 6, the values of phase differences normalized by the wave number change little in the Froude number range $0.20 < F_n < 0.27$ although, as Hogben shows, these phase differences are slightly less for higher Froude numbers.

In the nonlinear theory the principle of superposition does not work, as is well known. The present results, however, derive from a nonlinear analysis of each

elementary wave. The elementary waves form the bow or stern wave by superposition. When the regular wave integral is evaluated by the stationary phase method, the wave usually is taken to be the superposition of two wave systems, transverse and divergent. However, because the divergent waves are known to be of almost zero amplitude near the ship hull, only transverse waves are significant. In this sense, the present evaluation of rays near the hull surface is meaningful. At some distance from the hull, the predicted approximate behavior of ray paths and phase differences help one to understand discrepancies between the predictions of linear theory and the experimental results.

The effect of nonlinear amplitude interaction on reflecting rays has been tested numerically as shown in Figure 7. The amplitude interaction cannot obstruct the reflection of rays from the ship surface. Reflecting rays are pushed outboard due to the amplitude interaction, causing the shape of the second caustic to change considerably. The paths of rays and reflected rays are highly dependent on the hull form through the nonuniform flow and wave amplitude interaction effects. In addition, near the caustic even the wave-induced flow field interacts with the wave propagation. Therefore, it may be quite difficult to verify quantitatively the location of the second caustic observed in the towing tank. Nevertheless, the influence of the wave-induced velocity near the second caustic can be considered in the following way.

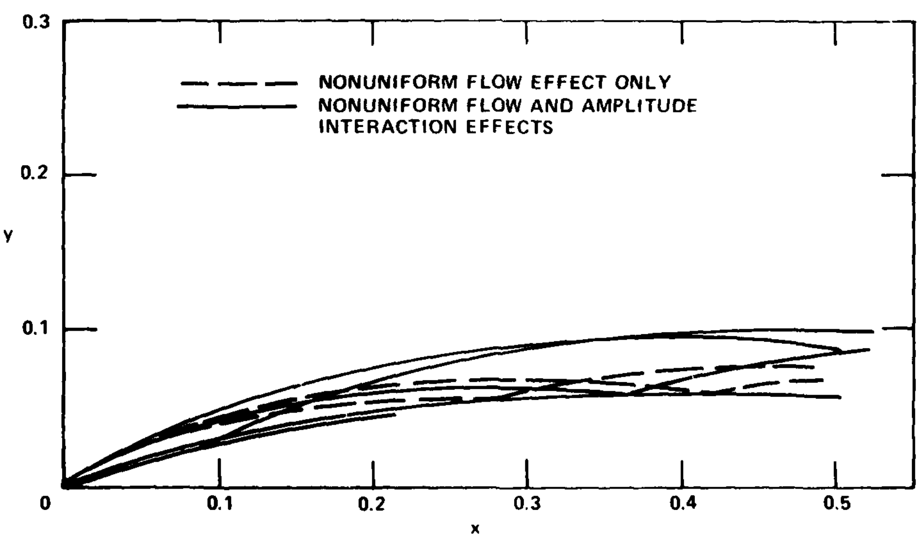

Figure 7 - Amplitude Interaction on Ray Reflections from the Wigley Hull
($b=0.2$, $h=0.0625$, $F_n=0.2$)

The ray equations which include the amplitude interaction can be obtained from Equation (24) by neglecting the quadratic amplitude terms:

$$\frac{d\theta}{dt} = 2 \sin\theta \left(\cos\theta \frac{\partial u}{\partial x} + \sin\theta \frac{\partial v}{\partial x}\right) - 2\left(\cos\theta \frac{\partial u}{\partial y} + \sin\theta \frac{\partial v}{\partial y}\right)$$

$$\frac{dx}{dt} = 2 \sin\theta\ (u \sin\theta - v \cos\theta) + \cos\theta\ (u \cos\theta + v \sin\theta)$$

The first equation can be written in a vector form

$$\frac{d\theta}{dt} \approx 2\bar{n} \cdot \frac{\partial \bar{q}}{\partial t_1} \tag{27}$$

where $\partial/\partial t_1$ is along the tangent to the crest of the wave, and the vectors are given by

$$\bar{t}_1 = \bar{i} \sin\theta - \bar{j} \cos\theta$$

$$\bar{n} = \bar{i} \cos\theta + \bar{j} \sin\theta$$

and

$$\bar{q} = u\bar{i} + v\bar{j}$$

Because the reflecting wave crest is almost perpendicular to the ship hull[9] and $\bar{q}$ near the ship is almost parallel to the ship hull, $\bar{n}$ and $\bar{q}$ are almost parallel, i.e., $\bar{n} \cdot \bar{q} \approx /\bar{q}/$. If the wave height increases, $/\bar{q}/$ will decrease according to linear wave theory. Therefore, if the ray crosses the second caustic into the large amplitude wave region, $d\theta/dt$ will be negative and θ will decrease according to Equation (27). Conversely, if the reflected ray crosses the second caustic out of the large amplitude wave region, $d\theta/dt$ will be positive and θ will increase. Then the reflected ray will go farther from the ship hull. This will cause the second caustic to occur farther from the ship hull, thus leaving a wider region of breaking

waves near the ship. Therefore, the actual breaking wave region will be larger than that predicted by the nonuniform flow effect only. The energy flux of the elementary bow waves near $\theta_\infty = 0$ that can reflect from a ship hull is limited to a certain range of the initial value of θ. The range depends upon the bow shape. If the bow shape is a wedge, there will be no reflection.[9] If it is a large round bow, there may be a large range of θ whose rays will reflect from the ship hull. However, near the stagnation point and the boundary layer the flow does not vary slowly enough for the present theory to be applied. Therefore, ships of large beam length ratio have a large curvature near the bow and show in the towing tank a few of the distinctive phenomena predicted here. However, for a thin model in a towing tank, the reflected wave energy may be too weak, and the second caustic be too close to the ship hull to overcome the boundary layer effect. In this case linear theory may predict rather well. To conclude, the nonlinear effects of amplitude interaction on ray paths and phases of ship waves have been investigated using the Stokes[1] nonlinear wave formula and the Whitham[5] nonlinear wave theory in conjunction with ray theory. For the ray path the nonuniform flow effect is greater than the effect of the amplitude interaction, but for the phase the latter effect is greater than the former.

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APPENDIX A
LINEAR WAVE AMPLITUDE

In the ray equation the computed value of the linear wave amplitude a is used. The height of the regular bow wave caused by the source distribution m for the Wigley hull is

$$\zeta = 4 \int_{-\pi/2}^{\pi/2} \int_{-h}^{0} \int_{0}^{1} m(x_1, z_1)\, e^{k_o z_1 \sec^2\theta + i k_o \sec^2\theta\{(x-x_1)\cos\theta + y\sin\theta\}}\, k_o \sec^3\theta \, dx_1\, dz_1\, d\theta$$

where

$$k_o = gL/U^2 = F_n^{-2}$$

$$m(x_1, z_1) = b_1 (1 - x_1{}^2) \left\{ 1 - \left(\frac{z_1}{h}\right)^2 \right\}$$

then

$$\zeta = 4 b_1 \int_{-\pi/2}^{\pi/2} E_1 \left(\frac{2}{k_o \sec^2\theta} + i \right) e^{i k_o (x \sec\theta + y \sec^2\theta \sin\theta)}\, d\theta$$

$$E_1 = \frac{1}{k_o} - \frac{2}{h^2 k_o{}^3 \sec^4\theta} + \frac{2 e^{-k_o h \sec^2\theta}}{h k_o{}^2 \sec^2\theta} \left(1 + \frac{1}{k_o h \sec^2\theta} \right)$$

Using the method of stationary phase, the amplitude can be obtained as

$$a = 4 b_1 E_1 \left\{ \left(\frac{4}{k_o{}^2 \sec^2\theta_1} + 1 \right) \left(\frac{2\pi}{k_o r |F''(\theta_1)|} \right) \right\}^{1/2}$$

where

$$x = r \cos \delta, \; y = r \sin \delta$$

$$F(\theta) = \cos \delta \sec \theta + \sin \delta \sec^2 \theta \sin \theta$$

θ_1 satisfies the equation

$$2 \tan^2 \theta_1 + \cot \delta \tan \theta_1 + 1 = 0$$

or

$$\tan \delta = - \tan \theta_1/(2 \tan^2\theta_1+1)$$

and

$$F''(\theta_1) = \sec^3 \theta_1(1-2 \tan^2\theta_1) \{(2 \tan^2\theta_1+1)+ \tan^2\theta_1\}^{-1/2}$$

$$a_\theta = 4\left[\frac{\partial E_1}{\partial \theta_1}\left(\frac{4}{k_o^2\sec^2\theta_1}+1\right)^{1/2} + 2E_1\left(\frac{4}{k_o^2\sec^2\theta_1}+1\right)^{-1/2}\right]\left(\frac{2\pi}{k_o r|F''(\theta_1)|}\right)^{1/2}$$

$$- 2 E_1\left(\frac{4}{k_o^2\sec^2\theta_1}+1\right)^{1/2}\sqrt{\frac{2\pi}{k_o r}}\,|F''(\theta_1)|^{-3/2}\; F'''(\theta_1)$$

where

$$F'''(\theta) = \frac{\sec^3 \theta\{(2 \tan^2\theta+1)(3 \tan \theta+1)-4 \tan \theta(3 \tan^2\theta+\sec^2\theta)\}}{\sqrt{(2 \tan^2\theta+1)^2 + \tan^2 \theta}}$$

$$\frac{\partial E_1}{\partial \theta} = \frac{8}{h^2 k_o^3} \frac{\tan\theta}{\sec^4\theta} - \frac{4 e^{-k_o h \sec^2\theta} \tan\theta}{k_o} \left(1 + \frac{\cos^2\theta}{k_o h}\right)^2$$

$$+ \frac{4 e^{-k_o h \sec^2\theta}}{k_o^3 h^2} \cos^4\theta \tan\theta$$

$$a_x = a_\theta \frac{\partial\theta}{\partial x} + \frac{\partial a}{\partial x}$$

$$a_y = a_\theta \frac{\partial\theta}{\partial y} + \frac{\partial a}{\partial y}$$

where only r is considered to be a function of x or y in $\partial a/\partial x$ and $\partial a/\partial y$.

APPENDIX B

FLOW FIELD OF A WIGLEY HULL

For the computation of ray paths of a Wigley hull, the flow velocity and its derivatives u, v, u_x, v_y, u_y are needed on (x,y,o). A Wigley hull has the double model source distribution

$$m = b_1(-2x_1+1)\left\{1-\left(\frac{z_1}{h}\right)^2\right\}$$

in

$$0 < x_1 < 1,\ y_1 = 0,\ h > z_1 > -h$$

Thus,

$$-u\ (x,y,o) = 2\int_0^h\int_0^1 m\frac{\partial}{\partial x}\left(\frac{1}{r}\right)dx_1\ dz_1 - 1$$

$$= -2\int_0^h\int_0^1 m\frac{\partial}{\partial x_1}\left(\frac{1}{r}\right)dx_1\ dz_1 - 1$$

$$= 2\ b_1\left\{\int_0^h \frac{1-\frac{z^2}{h^2}}{r(x_1=1)}dz_1+\int_0^h \frac{1-\frac{z_1^2}{h^2}}{r(x_1=0)}dz_1\right\} - 1$$

$$-4\ b_1\left[\frac{2}{3}h\log\{x_1-x+r(z_1=h)\} - \int_0^h \frac{(x_1-x)\ y^2}{(z_1^2+y^2)\ r}dz_1\right.$$

$$\left.+\int_0^h\frac{(x_1-x)}{r}dz_1 - \frac{1}{3\ h^2}\int_0^h y^4\ \frac{(x_1-x)}{(z_1^2+y^2)\ r} - \frac{(x_1-x)(z_1^2-y^2)}{r}\ dz_1\right]_{x_1=0}^{x_1=1}$$

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$$- v\ (x,y,o) = 2\int_0^h \int_0^1 m \frac{\partial}{\partial y}\left(\frac{1}{r}\right) dx_1\ dz_1$$

$$= -2\ b_1 y \int_0^h \left[\frac{2x^2-3x+1}{(z_1{}^2+y^2)r(x_1=1)} \left(1+ \frac{y^2}{h^2}\right) - \frac{2x^2-3x+1}{r(y_1=1)h^2} - \frac{(2x^2-x)}{(z_1{}^2+y^2)r(0)} \left(1+ \frac{y^2}{h^2}\right) \right.$$

$$\left. - \frac{2z_1{}^2}{r(x_1=1)h^2} + \frac{2}{r(x_1=1)} + \frac{2x^2-x+2z_1{}^2}{r(x_1=0)h^2} - \frac{2}{r(x_1=0)} \right] dz_1$$

$$- u_x = -4\ b_1\ \log\ \left|\frac{r(x_1=1, z_1=0)}{r(x_1=0, z_1=0)}\right|$$

$$+ \frac{4}{h^2}\left[- \left(\frac{1}{2}\ z_1 r(x_1=1) - \frac{a_1{}^2}{2}\ \log\ |z_1+r(x_1=1)|\right) \frac{(x-1)}{a_1{}^2}\right.$$

$$+ 2\left\{\left(\frac{z_1}{2} + \frac{a_1{}^2}{4}\right)\log\ |z_1+r(r_1=1)| - \frac{z_1}{4}\ r(x_1=1)\right\}$$

$$- \frac{x}{a_0{}^2}\left(\frac{1}{2}\ z_1 r(x_1=0) - \frac{a_0{}^2}{2}\ \log\ |z_1+r(x_1=0)|\right)$$

$$\left. - 2\left\{\left(\frac{z_1{}^2}{2} + \frac{a_0{}^2}{4}\right) \log\ |z_1+r(x_1=0)| - \frac{z_1}{4}\ r(x_1=0)\right\} \right]_{z_1=0}^{h}$$

where

$$a_1{}^2 = (x-1)^2 + y^2$$

$$a_0{}^2 = x^2 + y^2$$

$$- u_y\ (x,y,o) = 2\int_0^h \int_0^1 m \frac{\partial^2}{\partial y \partial x}\left(\frac{1}{r}\right) dx_1\ dz_1$$

$$= -2\ b_1 y\left[\frac{h}{\{(1-x)^2+y^2\}r(1,h)} + \frac{1}{r(1,h)h} + \frac{1}{r(0,h)h}\right.$$

$$\left. - \frac{1}{h^2} \log \frac{\{h+r(1,h)\}\{h+r(0,h)\}}{r(1,0)\cdot r(0,0)} + \frac{h}{(x^2+y^2)r(0,h)}\right]$$

$$+ 4\ b_1 y\left\{\int_0^h \frac{(1-x)\left(1+\frac{y^2}{h^2}\right)}{(z_1{}^2+y^2)r(x_1=1)} dz_1 + \int_0^h \frac{x\left(1+\frac{y^2}{h^2}\right)}{(z_1{}^2+y^2)r(x_1=0)} dz_1\right.$$

$$\left. - \frac{1}{h^2}\int_0^h \frac{1-x}{r(x_1=1)} dz_1 - \frac{1}{h^2}\int_0^h \frac{x}{r(x_1=0)} dz_1\right\}$$

$$- v_y = b_1\left(6y^2 + \frac{6y^4}{h^2}\right)\left[\frac{1}{3}(1-2x)\left(\frac{1}{x_1-x} + \frac{(x_1-x)^2-y^2}{(x_1-x)y^2}\right)\right.$$

$$\times \int_0^h \frac{dz_1}{(z_1{}^2+y^2)r} + \frac{2}{3}\frac{h}{\{(x_1-x)^2+y^2\}r(z_1=h)}$$

$$+ \frac{1}{3}(1-2x)\left(\frac{-h}{(x_1-x)\{(x-x_1)^2+y^2\}r(z_1=h)}\right.$$

$$\left.\left. + \frac{hr(z_1=h)}{(h^2+y^2)(x_1-x)y^2}\right)\right]_{x_1=0}^{1} -6\ b_1 y^2\left[\frac{1}{3h^2}\left(\frac{2y^2+(1-2x)(x_1-x)h}{\{(x_1-x)^2+y^2\}r(z_1=h)}\right.\right. + \qquad \text{(cont.)}$$

$$+ 2 \log \frac{h+r(z_1=h)}{r(z_1=0)} - \frac{2h}{r(z_1=h)}\Bigg) + \frac{2(x_1-x)(1-2x)}{3h^2} \int_0^h \frac{dz_1}{(z_1^{\,2}+y^2)\,r}\Bigg]_{x_1=0}^{1} - \frac{v(x,y,o)}{y}$$

where

$$r(a,b) = r(x_1=a,\ z_1=b)$$

In these expressions the integrals

$$\int \frac{z_1^{\,n}}{r}\, dz_1$$

$$\int \frac{dz_1}{(z_1^{\,2}+y^2)\,r}$$

are known in closed form.

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